adopted for its reproduction, and also to ask whether a cheap reproduction of the star-charts alone (without the Milky Way) would be considered of any value for observatory use. I certainly found that the existence of such charts would have saved me a considerable amount of time and labour. May I, therefore, say that I should be very grateful for any general or special criticism, as well as for advice with regard to the best way to publish the maps?

A Discussion of Greenwich North Polar Distances of Polaris and other Stars, with reference to Corrections for Temperature and Humidity. By W. Grasett Thackeray.

(*Communicated by the Astronomer Royal.*)

The following paper is a continuation of one communicated to the Society last year, which will be published in vol. xlix. of the *Memoirs*, and is an endeavour to determine whether the discordances in the observed polar distances of *Polaris* arise from an imperfect correction for changes of temperature, or from the want of a correction for variations in the amount of humidity, or from some other cause.

The discordances here dealt with are the differences in the observed places of *Polaris* and *Polaris S.P.* as compared with the means of these observations grouped according to the readings of the exterior thermometer, or according to the value of humidity as given by the actual readings of the wet and dry bulb thermometer, taken from the photographic sheets, corresponding to the dates and times at which the observations of *Polaris* and *Polaris S.P.* were made.

The discordances for temperature depend on the observations made during the two periods, 1851–67, and 1877–84, and the discordances for humidity on the years 1877–84 only.

The following discordances, arranged according to temperature, are the result of a more careful discussion of the lower and higher temperatures, and an extension of the range of temperature in the latter direction, and represent errors of zenith distance.

Temperature	25°	30°·5	34°	38°	42°	46°	50°
1851–67 / 1877–84	+″·10$_{93}$	+″·16$_{111}$	+″·14$_{249}$	+″·18$_{393}$	+″·21$_{402}$	+″·06$_{436}$	+″·08$_{437}$
Temperature	54°	58°	62°	67°·5	72°·2	77°·0	83°·0
1851–67 / 1877–84	+″·04$_{366}$	−″·09$_{274}$	−″·16$_{274}$	−″·32$_{296}$	−″·45$_{130}$	−″·60$_{72}$	−″·49$_{21}$

Equating these residuals in the form $A + Bt =$ residual, where $t =$ temperature, and A and B constants,

$$140\,A + 6983\,B = +258''\cdot39,$$
$$6983\,A + 369721\,B = -\ 0''\cdot97;$$
$$\text{Whence } A = -0''\cdot721, \text{ and } B = +0''\cdot01432;$$

and assuming the value of the refraction for *Polaris* as 45″, our correction to refraction becomes

$$(1) \qquad \frac{\text{Refraction}}{45}(0\cdot0143\,t - 0\cdot721);$$

and the corrections corresponding to the above temperatures for *Polaris* are

Correction	−′·36	−″·26	−″·23	−″·18	−′·12	−″·07	−″·01
Residual	−·26	−·10	−·09	00	+·09	−·01	+·09
Correction	+·05	+·10	+·17	+·24	+·31	+·38	+·47
Residual	+·09	+·01	+·01	−·08	−·14	−·22	−·02

The following discordances depend on the observations of *Polaris* and *Polaris S.P.* made during the years 1877–84, and the values of relative humidity for each observation were determined as follows:—The readings of the dry and wet bulb thermometers were taken from the meteorological records for the nearest hour to the time of meridian passage on each day of observation, and the value of the corresponding relative humidity (= "h") was then obtained from Glaisher's Hygrometrical Tables. The observations were then arranged according to the values of "h" for each year both above and below pole, and the differences between the mean of each of these groups and the mean annual value found, and these differences for each year, both above and below pole, were afterwards combined, and are as follows, representing errors of zenith distance.

Value of "*h*"	98	92	87	82	77	72
1877–84	$+''\cdot21_{116}$	$+''\cdot26_{139}$	$+''\cdot24_{131}$	$+''\cdot08_{123}$	$+''\cdot08_{108}$	$+''\cdot02_{98}$
Value of "*h*"	68	62	58	53	47	43
1877–84	$-''\cdot14_{80}$	$-''\cdot29_{106}$	$-''\cdot31_{79}$	$-''\cdot47_{50}$	$-''\cdot66_{21}$	$-''\cdot81_{13}$

Equating these residuals in the form of $A + Bh =$ residual, where $h =$ humidity (saturation being equal to 100), we have

$$1043\,A + 80597\,B = -\ 5''\cdot24,$$
$$80597\,A + 6442707\,B = -3959''\cdot02.$$
$$\text{Whence } A = +1''\cdot274, \text{ and } B = -0''\cdot016558;$$

and our assumed correction for humidity becomes

$$(2) \qquad \frac{\text{Refraction}}{45}(+1\cdot27-0\cdot0166\,h);$$

and the corrections corresponding to the above humidities for *Polaris* are

Correction	−″·34	−″·24	−″·16	−″·08	″·00	+″·09
Residual	−·13	+·02	+·08	00	+·08	+·11
Correction	+·15	+·25	+·32	+·40	+·50	+·57
Residual	+·01	−·04	+·01	−·07	−·16	−·24

In order to compare the values for "h" and "t," the observations of 1877–84 were arranged according to temperature, and the values of "h" corresponding to this arrangement also found.

Temperature	25°	30°	34°	38°	42°	46°	50°
1877–84	$+''\cdot28_{16}$	$+''\cdot25_{36}$	$+''\cdot08_{59}$	$+''\cdot20_{131}$	$+''\cdot25_{147}$	$+''\cdot13_{141}$	$+''\cdot18_{136}$
"h"	82	84	87	85	84	82	81
Temperature	54°	58°	62°	67°	72°	77°	83°
1877–84	$+''\cdot08_{109}$	$-''\cdot36_{83}$	$-''\cdot28_{68}$	$-''\cdot45_{19}$	$-''\cdot60_{40}$	$-''\cdot78_{20}$	$-''\cdot64_{8}$
"h"	76	70	66	64	58	55	47

and the corresponding residuals, after applying our assumed corrections for "t" (1) and "h" (2) are

(1)	−″·08	−″·01	−″·15	+″·02	+″·13	+″·06	+″·17
(2)	+·20	+·14	−·08	+·07	+·14	+·05	+·12
(1)	+·13	−·26	−·11	−·21	−·29	−·40	−·17
(2)	+·10	−·24	−·09	−·23	−·26	−·41	−·14

The errors of the Sun's tabular N.P.D. for each month in the years 1877–86, corrected to one system of R — D and Bessel's refractions, and the present adopted value of colatitude for Greenwich, viz. 38° 31′ 21″·90, and furthermore reduced to represent the refractions referred to the Standard Meteorological thermometer, have been carefully computed. The means were also taken of the Standard thermometers corresponding to each group of observations to enable us to obtain a value for "t."

To obtain the value of "h" the values of the dry and wet bulb thermometer were taken from the meteorological records for each day of observation, and the corresponding value of relative humidity ("h") was taken from Glaisher's tables, and the mean of each group taken.

	Jan.	Feb.	March.	April.	May.	June.
Tabular Errors 1877–86	+″·20	−″·20	−″·39	+″·02	+″·25	+″·24
Approx. Mean N.P.D.	111°	103°	92°	80°	72°	67°
„ Mean Refraction	190″	120″	80″	50″	40″	33″
	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Tabular Errors 1877–86	+″·58	+″·33	+″·19	+″·18	+″·04	−″·26
Approx. Mean N.P.D.	68°	76°	87°	99°	108½°	113°
„ Mean Refraction	34″	45″	65″	100″	160″	210″
Temperature	38°·5	44°·0	51°·5	55°·1	63°·0	77°·0
Humidity	75	74	65	61	55	54
Temperature	72°·6	71°·2	65°·0	54°·7	45°·8	39°·2
Humidity	54	57	62	69	73	78

The residuals, after applying the corrections for "t" (1) and "h" (2), corresponding to the above temperatures and humidities are

(1)	+″·91	+″·03	−″·46	−″·03	+″·09	−″·06	+″·34	+″·03	−″·11	+″·08	+″·25	+″·44
(2)	+″·08	−″·34	−″·74	−″·25	−″·08	−″·06	+″·28	00	−″·16	−″·12	−″·21	−″·22

As these results were not satisfactory, and gave no assurance that either correction was an unmixed advantage, a table was made of the corrections for every 10° of zenith distance corresponding to what we assumed would be the extreme ranges of temperature and humidity for night observations, in order to see what sort of discordances might be expected to show themselves in the observations of the stars. The extreme temperatures were assumed to be 65° and 25°, and the extreme humidities 100 and 50. The following is the table:—

Zen. Dist.	Approx. Mean Refraction.	Temperature. 65°	Temperature. 25°	Relative Humidity. 100	Relative Humidity. 50
°	″	″	″	″	″
60	100	+0·5	−0·7	−0·8	+0·9
70	160	0·7	1·3	1·3	1·5
80	320	1·5	2·5	2·6	3·1
81	350	1·7	2·8	2·9	3·5
82	390	1·9	3·2	3·3	4·0
83	440	2·1	3·5	3·8	4·4
84	500	2·3	4·0	4·0	4·9
85	590	2·7	4·6	4·7	5·8
86	700	+3·2	−5·6	−5·6	+6·8

We then determined to see what effect these corrections would have on the observations of low southern stars, and the answer is apparently decisive. The observed places will allow of neither the one nor the other correction, but they show that the refractions of the Fundamenta give more satisfactory results than those of the Tabulæ, as also that any correction due to humidity is exceedingly small.

Taking the observations of low southern stars of Z.D. 82° 50′ and upwards for the years 1876–86, and correcting each daily result for R — D, colatitude and refraction, the same as for the mean places in the Greenwich 1880 Catalogue, and taking the assumed place, annual precession, and proper motion from the Cape 1880 Catalogue (Stone), we have called the difference between the observed places at Greenwich and the Cape, error of refraction at Greenwich. The mean Z.D. of all the stars used is 85°·½.

Taking the reading of the thermometer used in the reductions of each observation from the volume of Greenwich observations we have corrected these errors of refraction for our assumed law of correction for temperature (1) and arranged the errors in order of temperature, also giving the corrected errors for the corresponding humidities.

Taking the readings of the dry and wet bulb thermometers for the nearest half hour to the time of meridian passage of each star from the meteorological records we have found the relative humidities ("h") from these data, and corrected these errors of refraction according to our assumed law (2) for correction for humidity, and arranged the errors in order of humidity, also giving the corrected errors for the corresponding temperatures.

Finally we have diminished the log mean refractions of the Tabulæ for all observations above 85° Z.D. by ·00143 in order to show the errors of refraction as given by Bessel's Fundamenta, and the corresponding errors of refraction are given in each table.

Table I.

Temperature.		No. of Observations.		Error of Refraction (Tabulæ).		Error of Refraction (Tabulæ). Corrected for Temperature.		Error of Refraction (Tabulæ). Corrected for Humidity.		Error of Refraction. Fundamenta.	
°	°			″	″	″	″	″	″	″	″
31·5	35·3	11	31	+3·74	+2·65	+0·39	−0·57	+2·11	+0·16	+1·37	+0·68
37·4		20		+2·04		−1·10		−0·91		+0·30	
42·0	45·9	20	66	+3·00	+2·43	+0·29	+0·46	+1·16	+0·43	+1·38	+0·86
47·5		46		+2·21		+0·54		+0·11		+0·64	
52·7	55·5	49	95	+1·32	+1·53	+0·60	+1·63	−0·36	+0·33	−0·35	−0·18
56·8		32		+1·19		+1·49		−0·11		−0·34	
64·6		14		+2·96		+5·60		+3·72		+0·78	

Table II.

Humidity.		No. of Observations.		Error of Refraction (Tabulæ).		Error of Refraction (Tabulæ). Corrected for Humidity.		Error of Refraction (Tabulæ). Corrected for Temperature.		Error of Refraction. Fundamenta.	
				″	″	″	″	″	″	″	″
97	93	13	58	+2·47	+2·09	−2·07	−1·70	+1·13	+0·63	+0·90	+0·28
92		45		+1·98		−1·64		+0·49		+0·10	
86	85	52	87	+1·76	+2·10	−0·28	+0·43	+0·27	+0·72	+0·23	+0·50
82		35		+2·73		+1·64		+1·77		+0·91	
75	73	36	45	+1·59	+1·74	+2·53	+3·05	+1·12	+1·67	−0·31	−0·13
65		9		+2·31		+5·14		+3·89		+0·60	

The mean error of refraction (Tabulæ) given by the stars below 85° Z.D. is $+1''\cdot00$ depending on 72 observations.

We now assume that the discordances are not connected with the correction for refraction, that they are still due to temperature, and probably closely connected with changes in the instrument itself, and that they vary as the sin Z.D. Our correction for temperature then becomes

$$(3)\quad \frac{\sin z}{\sin 38^{\circ}\cdot31'}(0''\cdot0143\,t-0''\cdot721)=\sin z\,(0''\cdot0230\,t-1''\cdot158),$$

and the following table represents the corrections to the low southern stars:—

Temperature.		No. of Observations.		Error of Refraction (Tabulæ).		Error of Refraction. Fundamenta.		Tabulæ Corrected. $(-1''\cdot158+''\cdot0230\,t)\sin z$.		Fundamenta Corrected.	
°	°			"	"	"	"	"	"	"	"
31·5	35·3	11	31	+3·74	+2·65	+1·37	+0·68	+3·30	+2·28	+0·93	+0·32
37·4		20		+2·04		+0·30		+1·72		−0·02	
42·0	45·9	20	66	+3·00	+2·43	+1·38	+0·86	+2·80	+2·34	+1·18	+0·74
47·5		46		+2·21		+0·64		+2·12		+0·55	
52·7	55·5	49	95	+1·32	+1·53	−0·35	−0·18	+1·35	+1·62	−0·32	−0·08
56·8		32		+1·19		−0·34		+1·32		−0·21	
64·6		14		+2·96		+0·78		+3·26		+1·08	

The residuals, after applying this correction (3) to the errors of tabular N.P.D. of the Sun, as given above, are

January.	February.	March.	April.	May.	June.
+″·51	+″·03	−″·32	+″·02	+″·10	−″·20
July.	August.	September.	October.	November.	December.
+″·22	−″·06	−″·08	+″·18	−″·24	+″·06

The mean error for the year being now +″·01 instead of +″·10.

These results are more satisfactory, and seem to point to a probable explanation of R — D. The case seems to stand thus: here is a star constantly observed throughout the year, and the observations, when tabulated according to temperature, give marked discordances with a range of nearly 1″. First of all, assuming that these discordances were due to either a defective temperature correction, or to the want of a correction for humidity, it has been shown that the observations of low southern stars not only do not require such a correction, but apparently demonstrate that there is little or no fault to be found with the refractions in this respect; and, secondly, assuming that this temperature correction is also instrumental, and varies as sin Z.D., it is further shown that such a correction, applied to the observations of low southern stars, and to the tabular errors of the Sun, in both cases, gives satisfactory results. The same stars, observed both by direct view and also by reflexion from mercury, show large and varying discordances which are tabulated as R — D according to zenith distance, and are supposed to be applicable half to the R observation and half to the D observation, though there has never been any direct proof of the truth of this arrangement, and a correction is computed from a formula A + B sin z, when A and B are constants determined yearly from the observed R — D. Now these discordances in the direct observations of *Polaris* appear to also conform themselves to the same formula, and to be also applicable as corrections to other observations, and it would, therefore, be fair to assume that R — D was entirely an instrumental correction, and should contain a term depending on temperature. The importance of such a correction in the cases of the Sun and Moon, which are observed at very extreme temperatures in the course of the year, needs no demonstration.

The following table represents the mean Z.D.'s of each group of R — D stars for the year 1887, the mean thermometer reading used in the reductions, the correction required by our assumed temperature formula sin z (0·0460 t − 2·316) as applicable to R — D, and the difference between the observed value of R — D, and the value computed by the formula −″·06 + 1·365 sin z in use for that year.

The result of applying the assumed temperature correction is to diminish the sum of the squares of the residuals from 5″·56 to 4″·71.

Table of Assumed Temperature Correction to Groups of R − D, 1887.

Mean Z. D. South.	Mean Temp.	Assumed Temperature Correction.	Error of Ordinary formula.	Corrected Error.	Mean Z. D. South.	Mean Temp.	Assumed Temperature Correction.	Error of Ordinary Formula.	Corrected Error.
° ′	°	″	″	″	°	°	″	″	″
−67 12	43·1	+·31	−1·13	−·82	+12·11	43·0	−·07	−·19	−·26
65 27	42·7	+·31	−0·95	−·64	19·50	46·8	−·05	+·02	−·03
63 19	42·4	+·32	+·28	+·60	24·16	46·4	−·08	−·02	−·10
62 21	46·7	+·14	−·01	+·13	27·31	45·7	−·09	+·14	+·05
59 4	45·5	+·18	+·23	+·41	31·1	45·0	−·12	+·36	+·24
55 43	50·5	00	+·23	+·23	33·28	45·0	−·13	+·39	+·26
51 41	46·8	+·12	−·27	−·15	37·38	51·2	·00	−·02	−·02
47 30	39·6	+·33	−·02	+·32	40·58	46·4	−·13	−·24	−·37
41 18	48·4	+·07	−·17	−·10	43·38	43·0	−·23	−·01	−·24
35 28	45·8	+·11	−·11	·00	46·46	51·6	+·05	+·24	+·29
30 15	45·4	+·12	−·09	+·03	49·2	45·7	−·15	−·07	−·22
25 52	48·3	+·04	+·25	+·29	52·23	51·0	00	+·05	+·05
21 26	44·3	+·10	+·12	+·22	56·23	48·2	−·08	−·41	−·49
16 12	42·8	+·06	+·55	+·61	60·48	42·1	−·33	−·66	−·99
13 13	50·9	00	+·37	+·37	62·11	61·2	+·44	−·40	·00
− 9 59	45·3	+·04	−·46	−·42	65·5	34·5	−·68	+·86	+·18
					+67·13	52·4	+·08	−·62	−·54

The results we may sum up as follows :—The effect on refraction due to humidity, if any, is exceedingly small. A correction depending on temperature, and varying as sin z, will satisfy the discordances in the observations of *Polaris*, it will get rid of an annual variation, also apparently depending on temperature, in the tabular errors of the Sun; as far as it goes it will tend to bring in greater accordance the errors derived from low southern stars when arranged in order of temperature, and it will diminish the sum of the squares of the residuals of the R — D formula. Each of these results being a link in a chain partaking of the nature of circumstantial evidence.

Results of Double-Star Measures at Windsor, New South Wales, during the Years 1886, 1887, and 1888. By John Tebbutt.

This communication comprises all the double-star results obtained here during the years 1886, 1887, and 1888. All the measures were made with the eight-inch equatorial, except those of *α Centauri*, on July 27 and August 6, 1886, which were made with the $4\frac{1}{2}$-inch instrument. The column headed "Hour Angles" contains the hour-angles between which each set of measures was taken, and the last column gives the weight assigned from a consideration of the conditions under which the observations were made. 1 denotes an unusually bad condition, and 5 an unusually good one.